EE309 Advanced Programming Techniques for EE

Lecture 20: Block cipher

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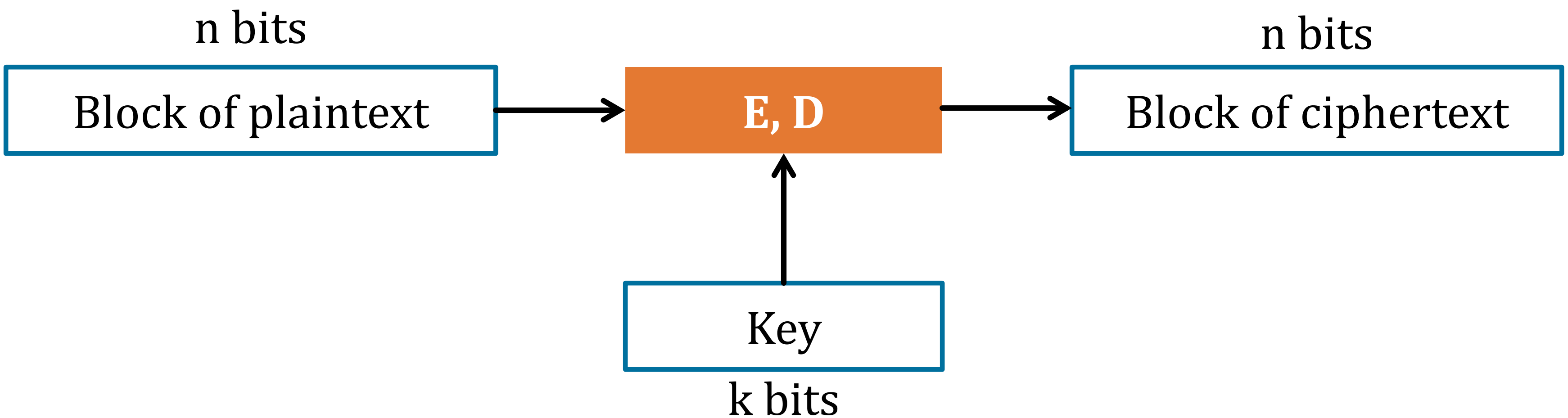
[Slides from 15-213: Introduction to Computer Systems at CMU]

What is a block cipher?

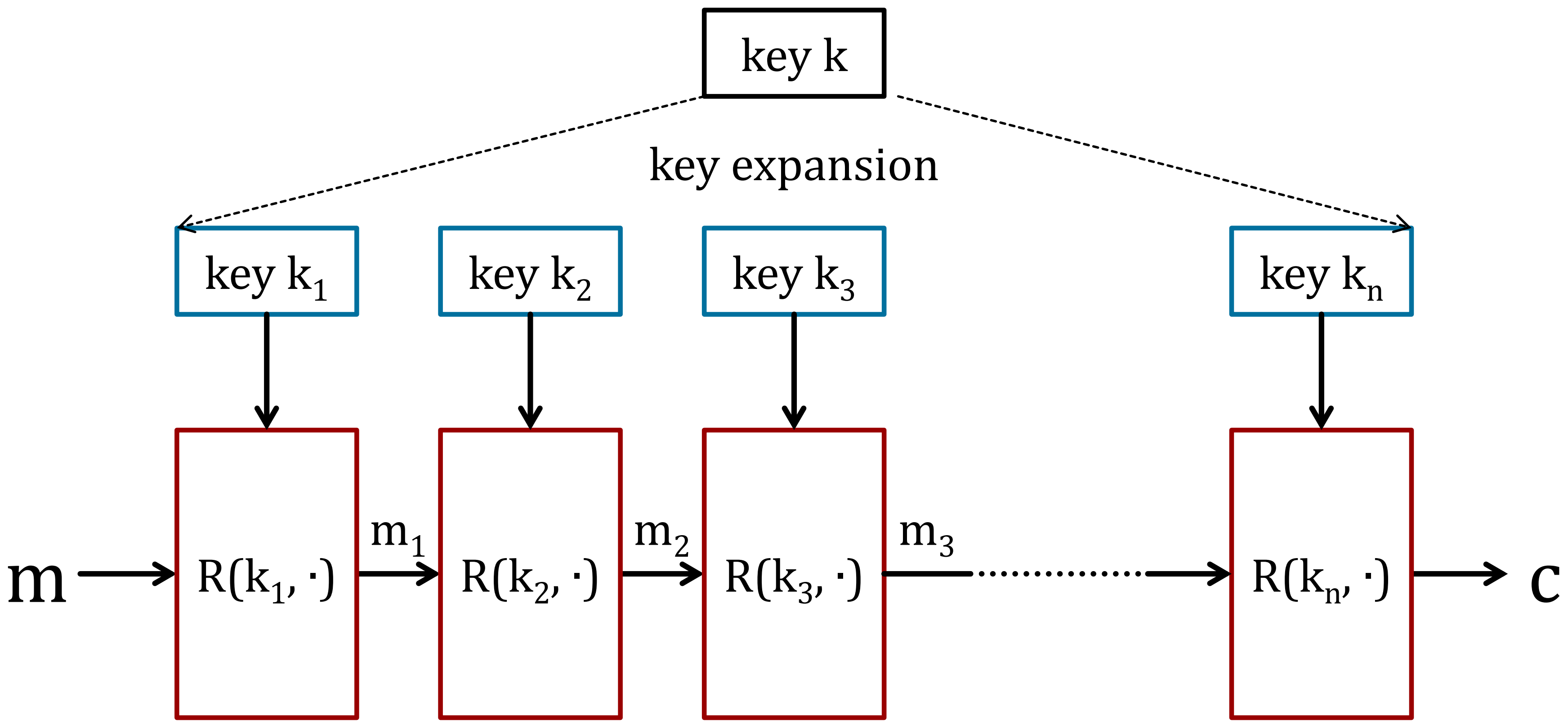
Block ciphers are the crypto work horse



Canonical examples:

1. 3DES: $n = 64$ bits, $k = 168$ bits
2. AES: $n = 128$ bits, $k = 128, 192, 256$ bits

Block ciphers built by iteration



R(k, m) is called a ***round function***

Ex: 3DES (n=48), AES128 (n=10)

Performance: Stream vs. block ciphers

Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	Cipher	Block/key size	Throughput [MB/s]
Stream	RC4		126
	Salsa20/12		643
	Sosemanuk		727
Block	3DES	64/168	13
	AES128	128/128	109

Block ciphers

The Data Encryption Standard (DES)

History of DES

- **1970s**: Horst Feistel designs Lucifer at IBM
  key = 128 bits, block = 128 bits

- **1973:** The National Bureau of Standards (NBS) asks for block cipher proposals.
  IBM submits variant of Lucifer.

- **1976:** NBS adopts DES as federal standard
  key = 56 bits, block = 64 bits

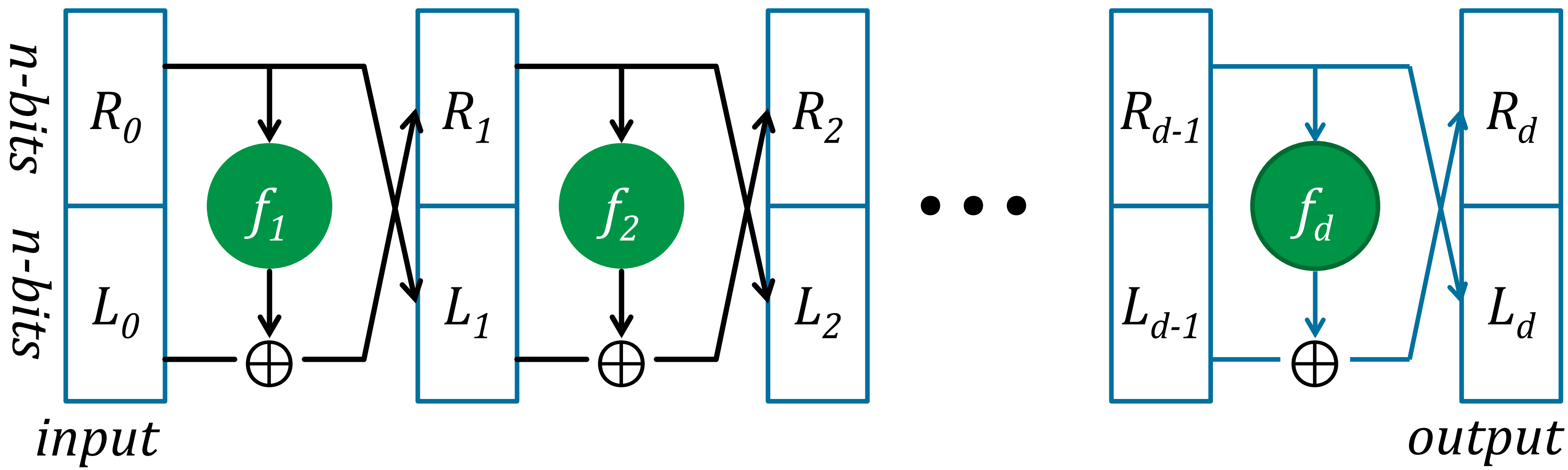
- **1997:** DES broken by exhaustive search

- **2000:** NIST adopts Rijndael as AES to replace DES. AES currently widely deployed in banking, commerce and Web

DES: core idea – Feistel network

Given one-way functions $f_1, \ldots, f_d : \{0,1\}^n \to \{0,1\}^n$

Goal: build invertible function $F : \{0,1\}^{2n} \to \{0,1\}^{2n}$



In symbols: $\begin{cases} R_i = f_i(R_{i-1}) \oplus L_{i-1} \\ L_i = R_{i-1} \end{cases}$

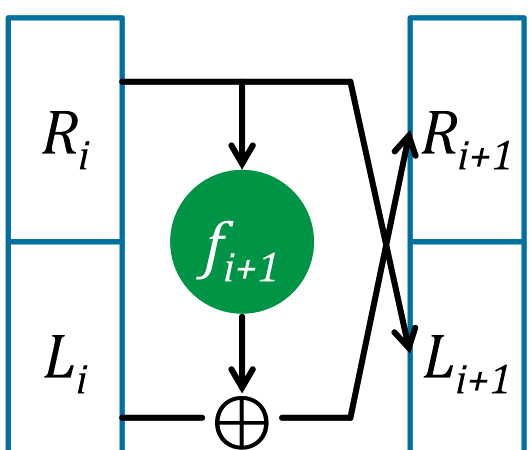
Feistel network - inverse

Claim: $f_1, \ldots, f_d : \{0,1\}^n \to \{0,1\}^n$

Feistel function *F* is invertible $F : \{0,1\}^{2n} \to \{0,1\}^{2n}$

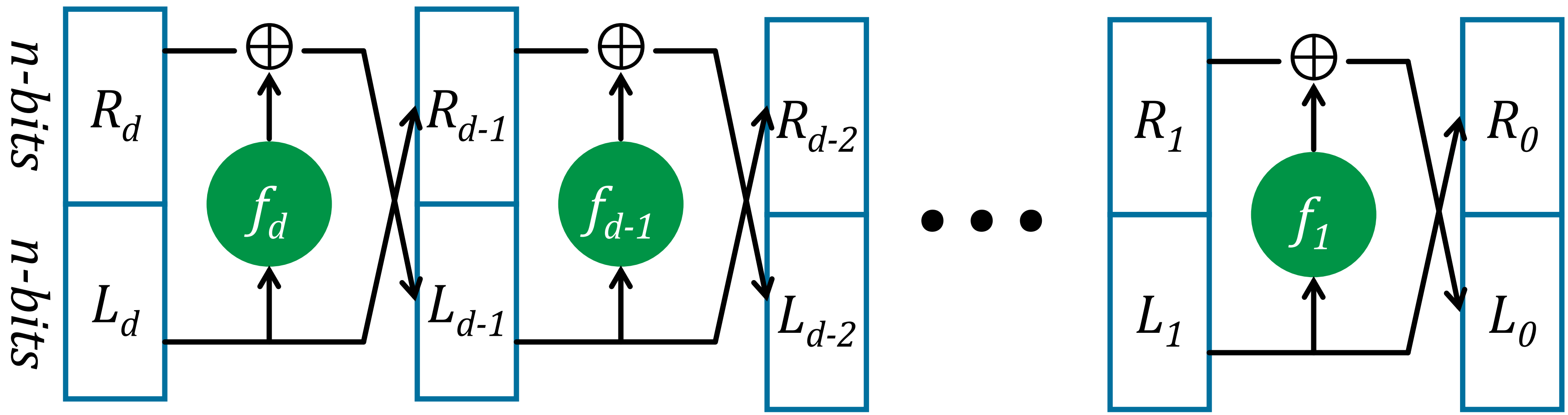
Proof: construct inverse

$$\begin{cases} R_i = \\ L_i = \end{cases}$$



inverse

Decryption circuit



- Inversion is basically the same circuit, with $f_1, \ldots, f_d$ applied in reverse order
- General method for building invertible functions (block ciphers) from arbitrary functions.
- Used in many block ciphers … but not AES

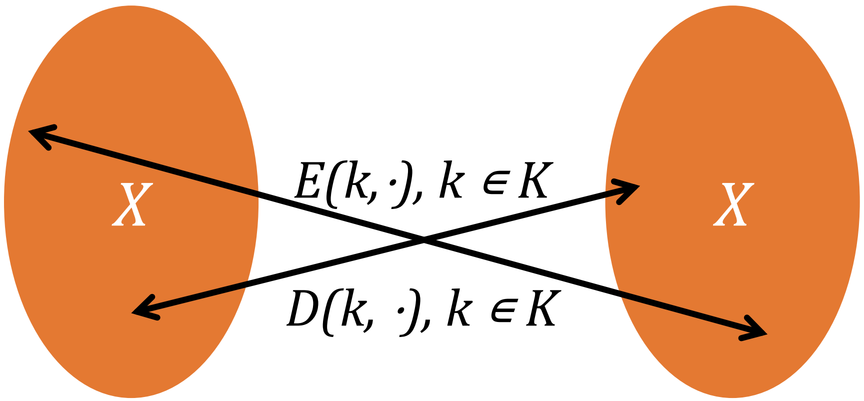
Recall from Last Time:
Block Ciphers are (Modeled As) PRPs

Pseudo Random Permutation (PRP) defined over (K,X)

$$E : K \times X \to X$$

such that:

1. Exists "efficient" deterministic algorithm to evaluate *E(k,x)*
2. The function *E(k, ·)* is one-to-one
3. Exists "efficient" inversion algorithm *D(k,y)*

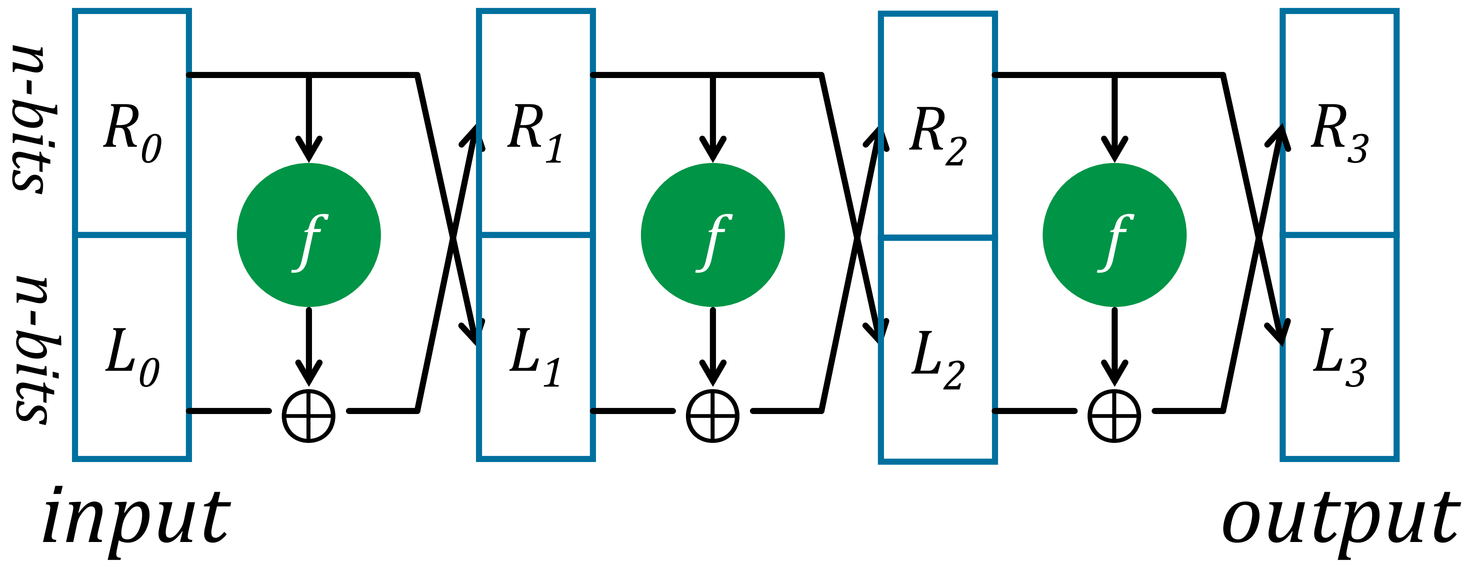


Luby-Rackoff Theorem (1985)

$f : K \times \{0,1\}^n \to \{0,1\}^n$ is a secure PRF

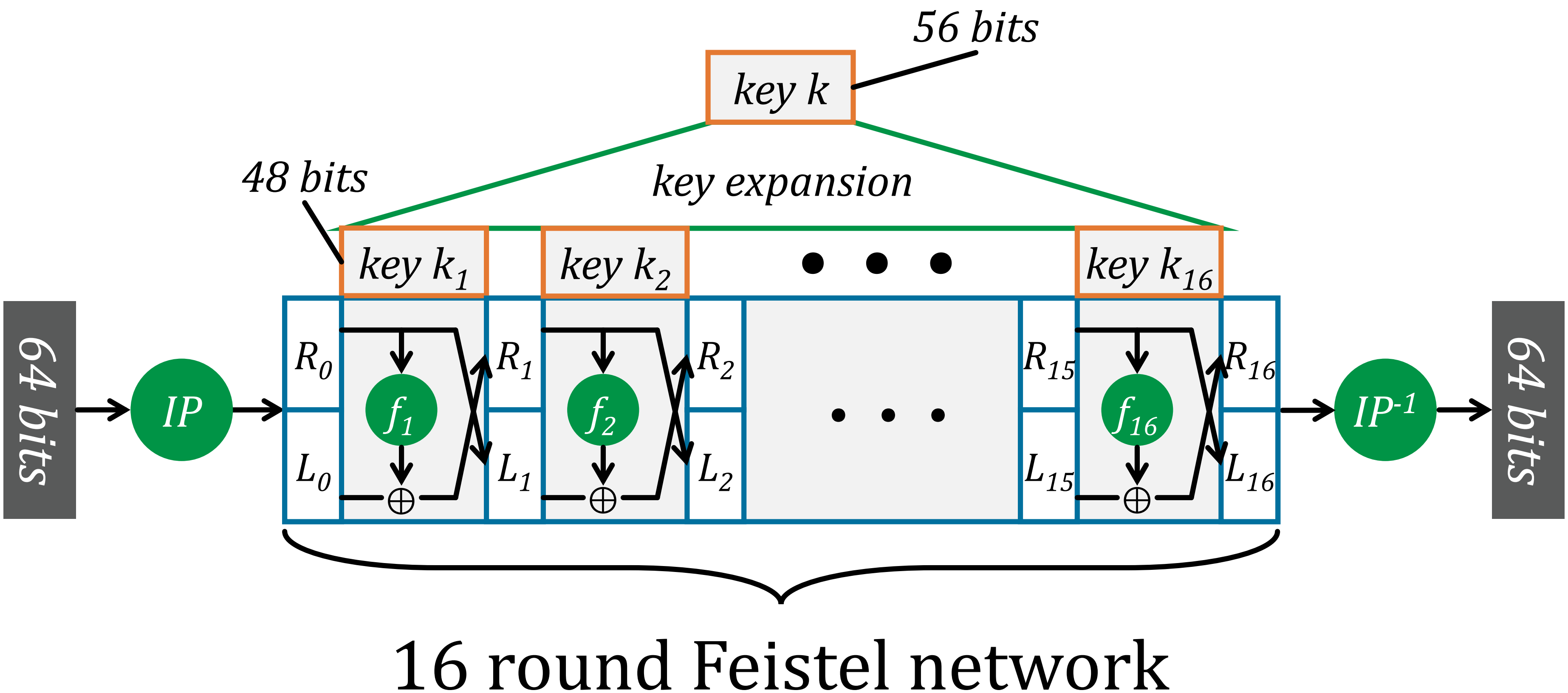
$\Rightarrow$ 3-round Feistel $F : K^3 \times \{0,1\}^{2n} \to \{0,1\}^{2n}$

is a secure PRP



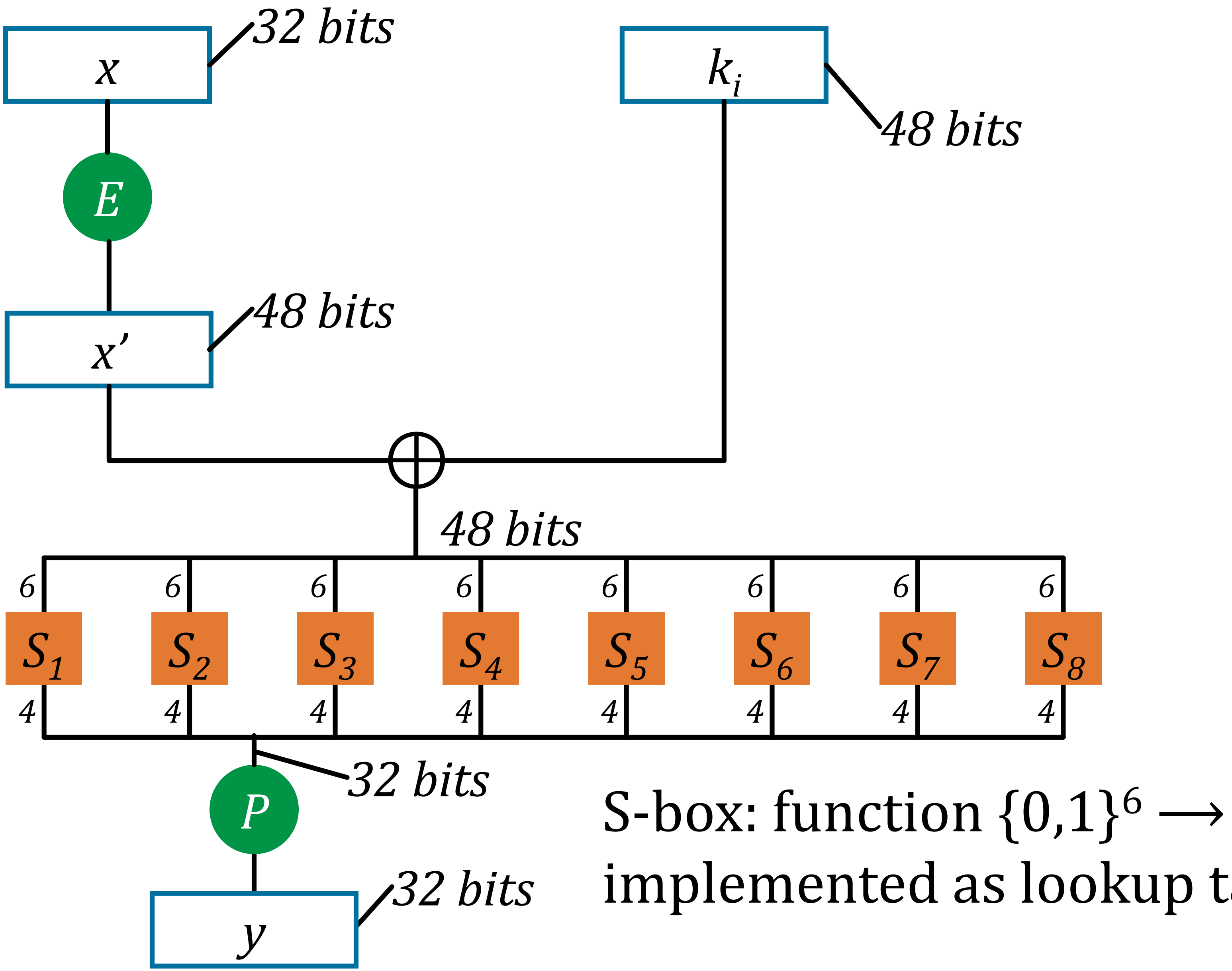
DES: 16 round Feistel network

$$f_1, \ldots, f_{16} : \{0,1\}^{32} \to \{0,1\}^{32} \text{ and } f_i(x) = \mathbf{F}(k_i, x)$$



To invert, use keys in reverse order

The function F(k_i, x)



S-box: function $\{0,1\}^6 \longrightarrow \{0,1\}^4$, implemented as lookup table.

The S-boxes

$$S_i : \{0,1\}^6 \rightarrow \{0,1\}^4$$

$$\text{e.g., } 011011 \longrightarrow 1001$$

S_5		Middle 4 bits of input															
		0000	**0001**	**0010**	**0011**	**0100**	**0101**	**0110**	**0111**	**1000**	**1001**	**1010**	**1011**	**1100**	**1101**	**1110**	**1111**
Outer bits	**00**	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

The S-boxes

- Alan Konheim (one of the designers of DES) commented, "We sent the S-boxes off to Washington. They came back and were all different.“
- 1990: (Re-)Discovery of differential cryptanalysis
  - DES S-boxes resistant to differential cryptanalysis!
  - Both IBM and NSA knew of attacks, but they were classified

Block cipher attacks

Exhaustive Search for block cipher key

Goal: given a few input output pairs

$(m_i, c_i = E(k, m_i))$ $i=1,..,n$ find key k.

Attack: Brute force to find the key k.

DES challenge

msg = "`The unknown messages is: XXXXXXXX`…"

msg =	"The unknown	messages is:	XXXXXXXX	…"
CT =	c_1	c_2	c_3	c_4

Goal: find $k \in \{0,1\}^{56}$ s.t. $DES(k, m_i) = c_i$ for $i=1,2,3$

Proof: Reveal $DES^{-1}(k, c_4)$

1976	DES adopted as federal standard		
1997	Distributed search	3 months	
1998	EFF deep crack	3 days	\$250,000
1999	Distributed search	22 hours	
2006	COPACOBANA (120 FPGAs)	7 days	\$10,000

$\Rightarrow$ 56-bit ciphers should not be used (128-bit key $\Rightarrow 2^{72}$ days)

Strengthening DES

Method 1: **Triple-DES**

Let $E : K \times M \longrightarrow M$ be a block cipher

Define **3E**: $K^3 \times M \longrightarrow M$ as:

$$\mathbf{3E}(\,(k_1,k_2,k_3),\, m) = \mathbf{E}(k_1,\, \mathbf{D}(k_2,\, \mathbf{E}(k_3,\, m)\,)\,)$$

3DES

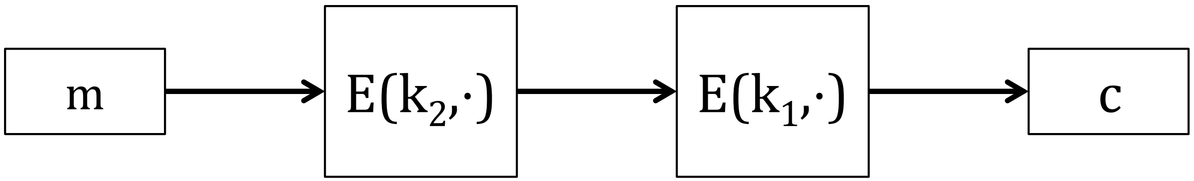
- Key-size: $3{\times}56 = 168$ bits
- $3\times$slower than DES
- Simple attack in time: $\approx 2^{118}$

$k_1 = k_2 = k_3 \Rightarrow$ DES

Why not 2DES?

- Define $\quad 2E(\,(k_1,k_2), m) = E(k_1\,, E(k_2\,, m)\,)$

key-len = 112 bits for 2DES

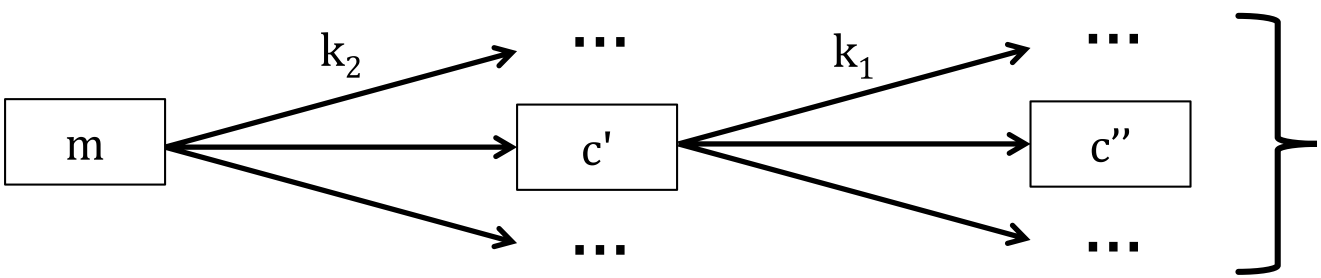


Naïve Attack: $M = (m_1,\ldots, m_{10})$, $C = (c_1,\ldots,c_{10})$.

For each $k_2 \in \{0,1\}^{56}$:

For each $k_1 \in \{0,1\}^{56}$:

if $E(k_2, E(k_1, m_i)) = c_i$ then (k_2, k_1)



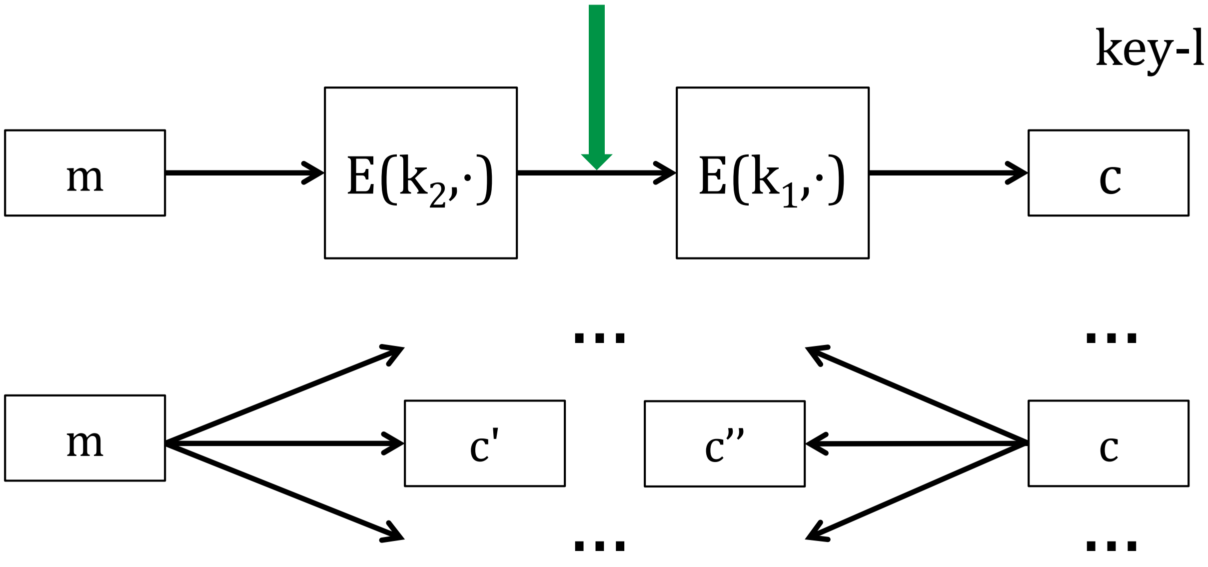
2^{112} checks

c'' = c?

Meet in the middle attack

- Define $2E(\,(k_1,k_2), m) = E(k_1\,, E(k_2\,, m)\,)$

key-len = 112 bits for 2DES

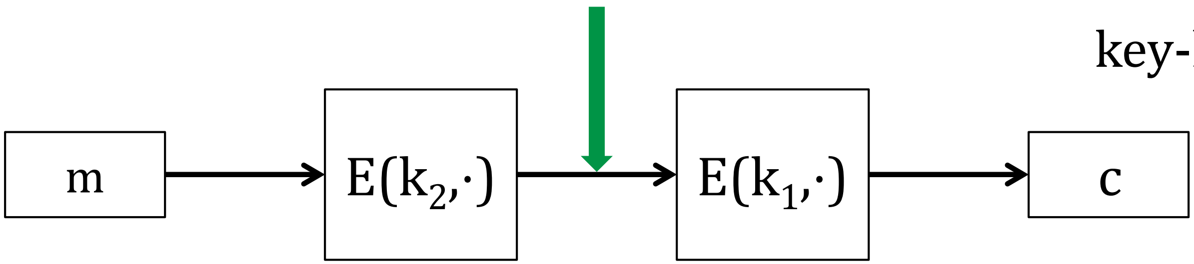


Idea: key found when c' = c'': $E(k_i, m) = D(k_j, c)$

Meet in the middle attack

- Define $2E(\,(k_1,k_2), m) = E(k_1\,, E(k_2\,, m)\,)$

key-len = 112 bits for 2DES



Attack: $M = (m_1,\ldots, m_{10})$, $C = (c_1,\ldots,c_{10})$.

- step 1: build table.

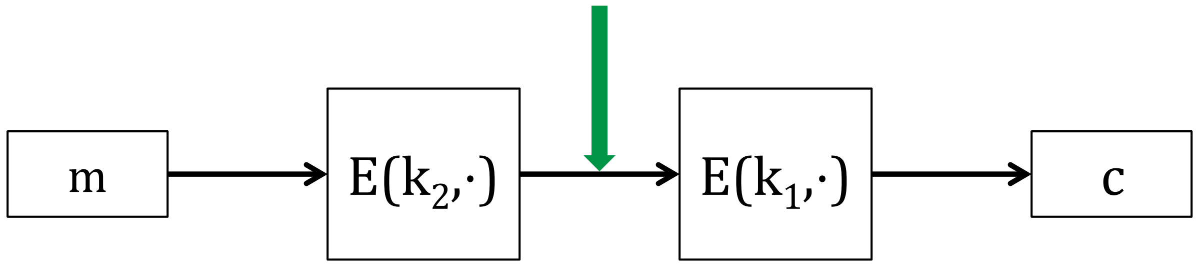
  sort on 2nd column

  maps c' to k_2

$k^0 = 00\ldots00$	$E(k^0, M)$
$k^1 = 00\ldots01$	$E(k^1, M)$
$k^2 = 00\ldots10$	$E(k^2, M)$
$\vdots$	$\vdots$
$k^N = 11\ldots11$	$E(k^N, M)$

2^{56} entries

Meet in the middle attack



$k^0 = 00\ldots00$	$E(k^0, M)$
$k^1 = 00\ldots01$	$E(k^1, M)$
$k^2 = 00\ldots10$	$E(k^2, M)$
$\vdots$	$\vdots$
$k^N = 11\ldots11$	$E(k^N, M)$

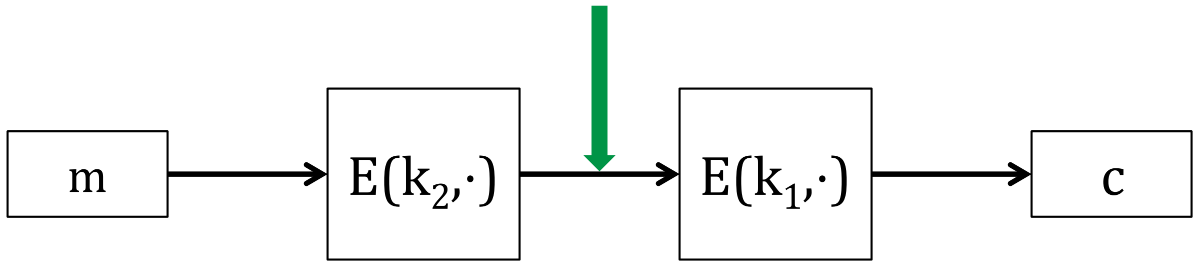
$M = (m_1, \ldots, m_{10})$, $C = (c_1, \ldots, c_{10})$

- step 1: build table.
- Step 2: for each $k \in \{0,1\}^{56}$:

  test if $D(k, c)$ is in 2nd column.

if so then $E(k^i, M) = D(k, C) \Rightarrow (k^i, k) = (k_2, k_1)$

Meet in the middle attack

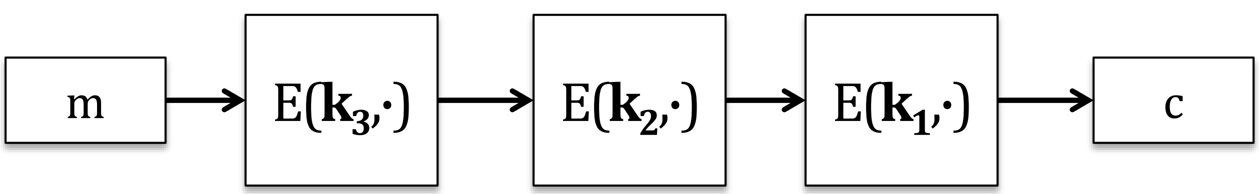


Time = $2^{56}\log(2^{56}) + 2^{56}\log(2^{56}) < 2^{63} \ll 2^{112}$

[Build & Sort Table] [Search Entries]

Space $\approx 2^{56}$ [Table Size]

Same attack on 3DES: Time = 2^{118}, Space $\approx 2^{56}$



Block ciphers

AES – Advanced encryption standard

The AES process

- 1997: DES broken by exhaustive search
- 1997: NIST publishes request for proposal
- 1998: 15 submissions
- 1999: NIST chooses 5 finalists
- 2000: NIST chooses Rijndael as AES
  (developed by Daemen and Rijmen at K.U. Leuven, Belgium)

Key sizes: 128, 192, 256 bits

Block size: 128 bits

AES core idea: Subs-Perm network

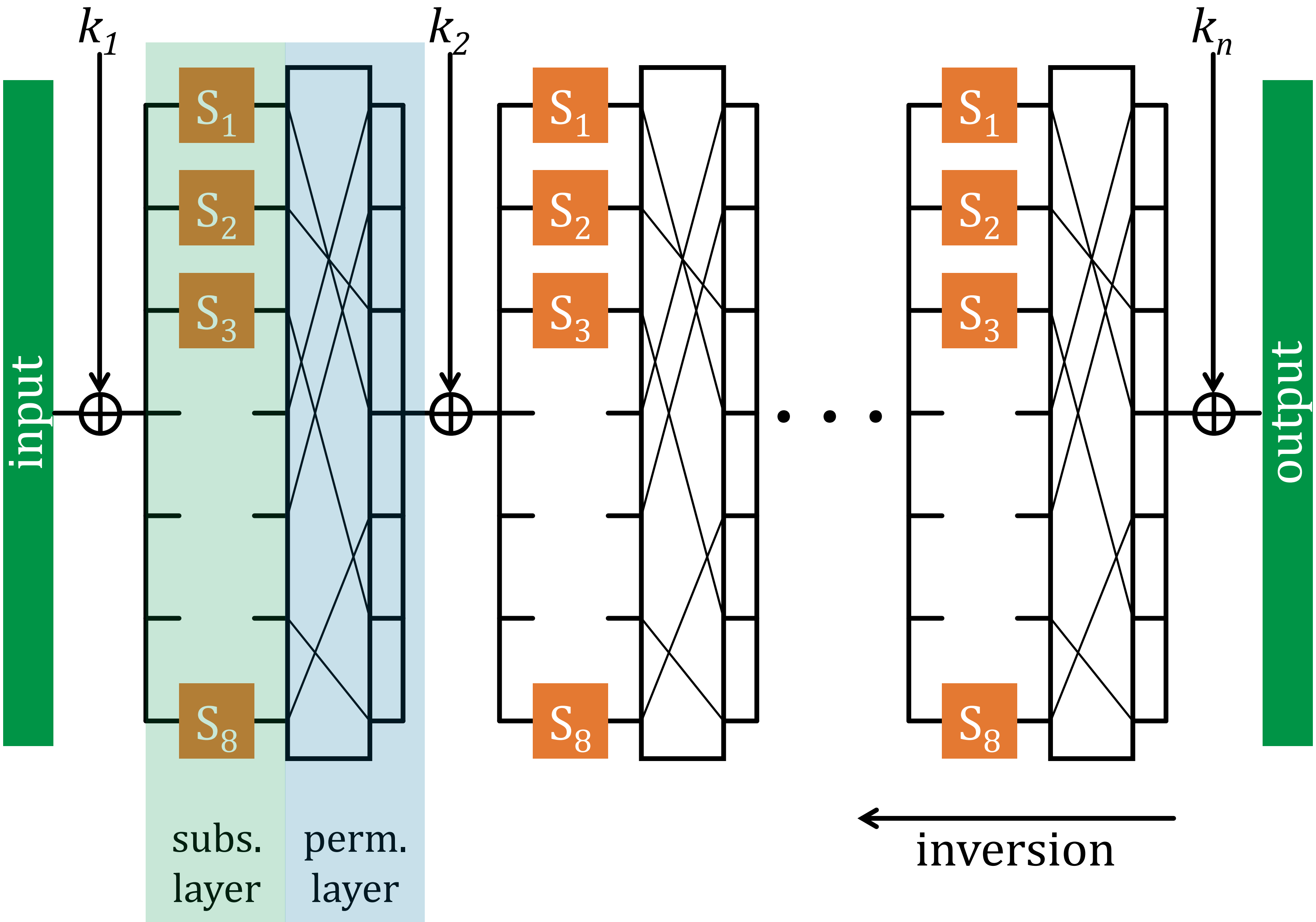
DES is based on Feistel networks

AES is based on the idea of

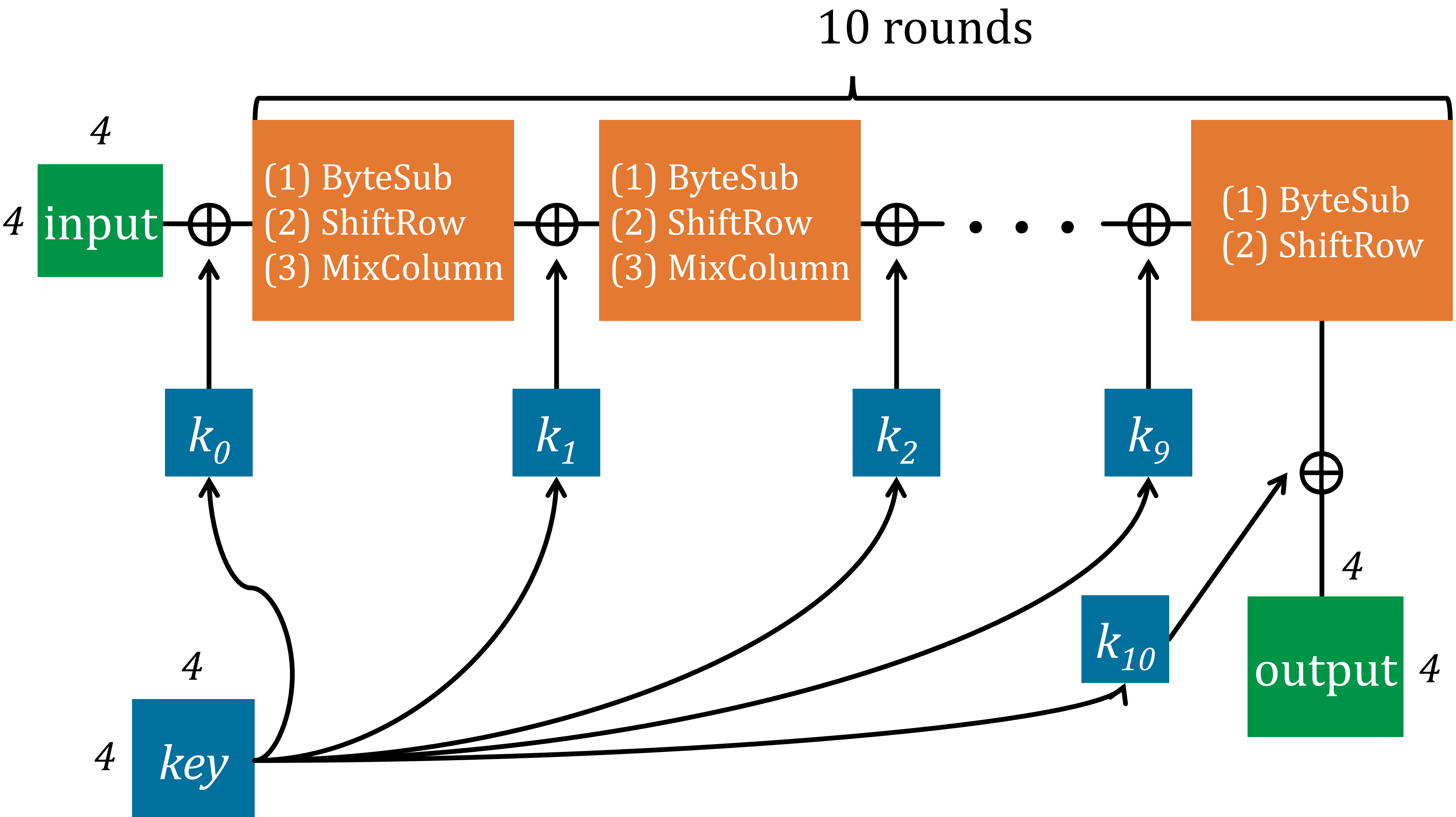
substitution-permutation networks

That is, alternating steps of substitution and permutation operations

AES: Subs-Perm network



AES128 schematic

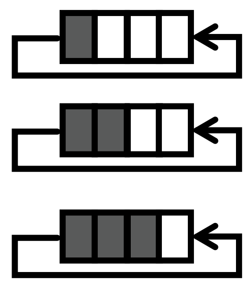


Key expansion: 16 bytes ⟶176 bytes

The round function

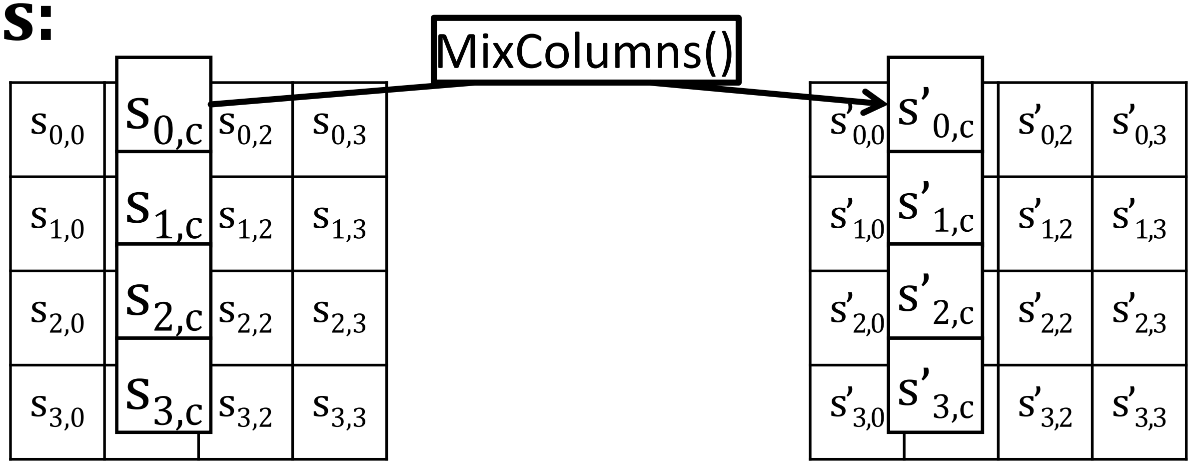
- **ByteSub:** a 1 byte *S*-box. 256 byte table (easily computable)
- **ShiftRows:**

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$



$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,0}$
$s_{2,2}$	$s_{2,3}$	$s_{2,0}$	$s_{2,1}$
$s_{3,3}$	$s_{3,0}$	$s_{3,1}$	$s_{3,2}$

- **MixColumns:**



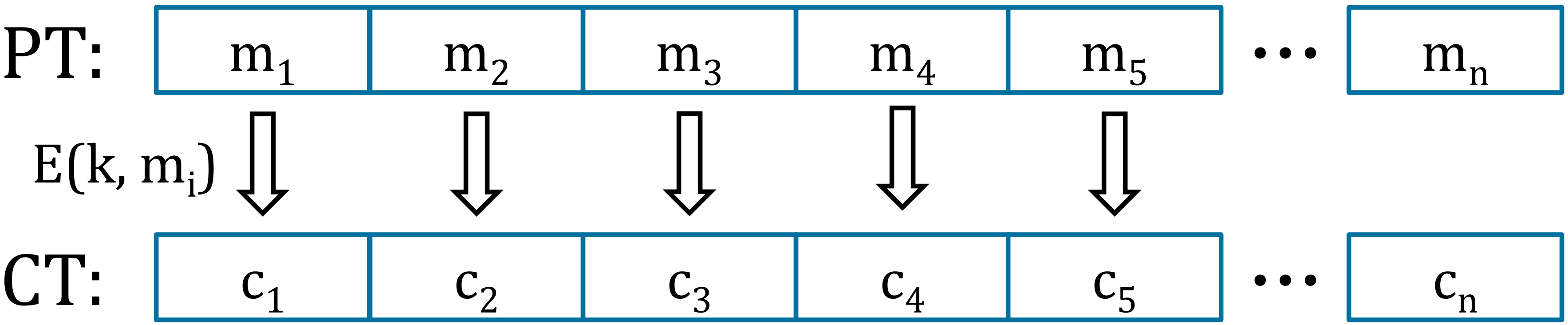
Security

- Many theoretical attacks have been proposed

- At present, there is **no known practical attack** that would allow someone without knowledge of the key to read data encrypted by AES when correctly implemented.

Modes of operation

Electronic Code Book (ECB) Mode



Problem:

$$m_1 = m_2 \longrightarrow c_1 = c_2$$

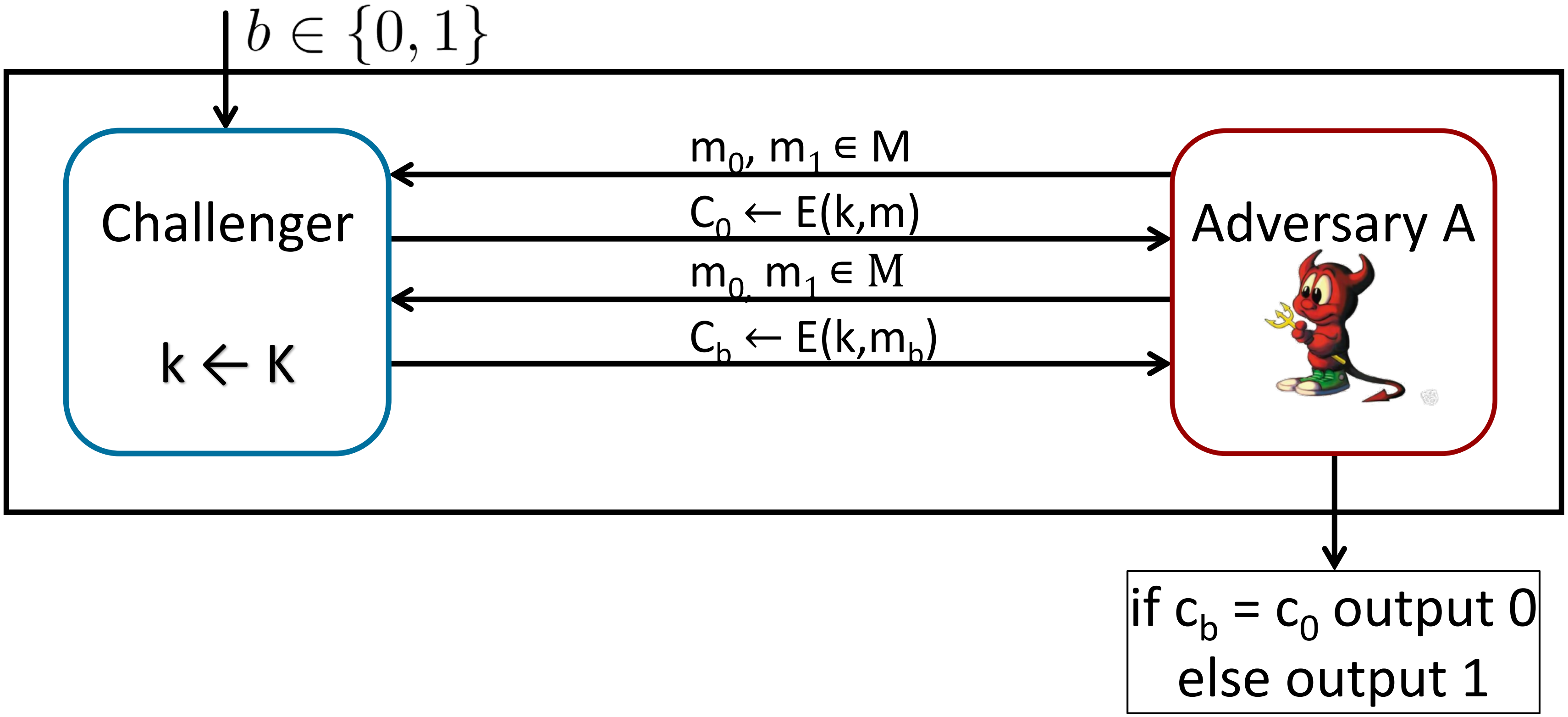
What can possibly go wrong?

Plaintext

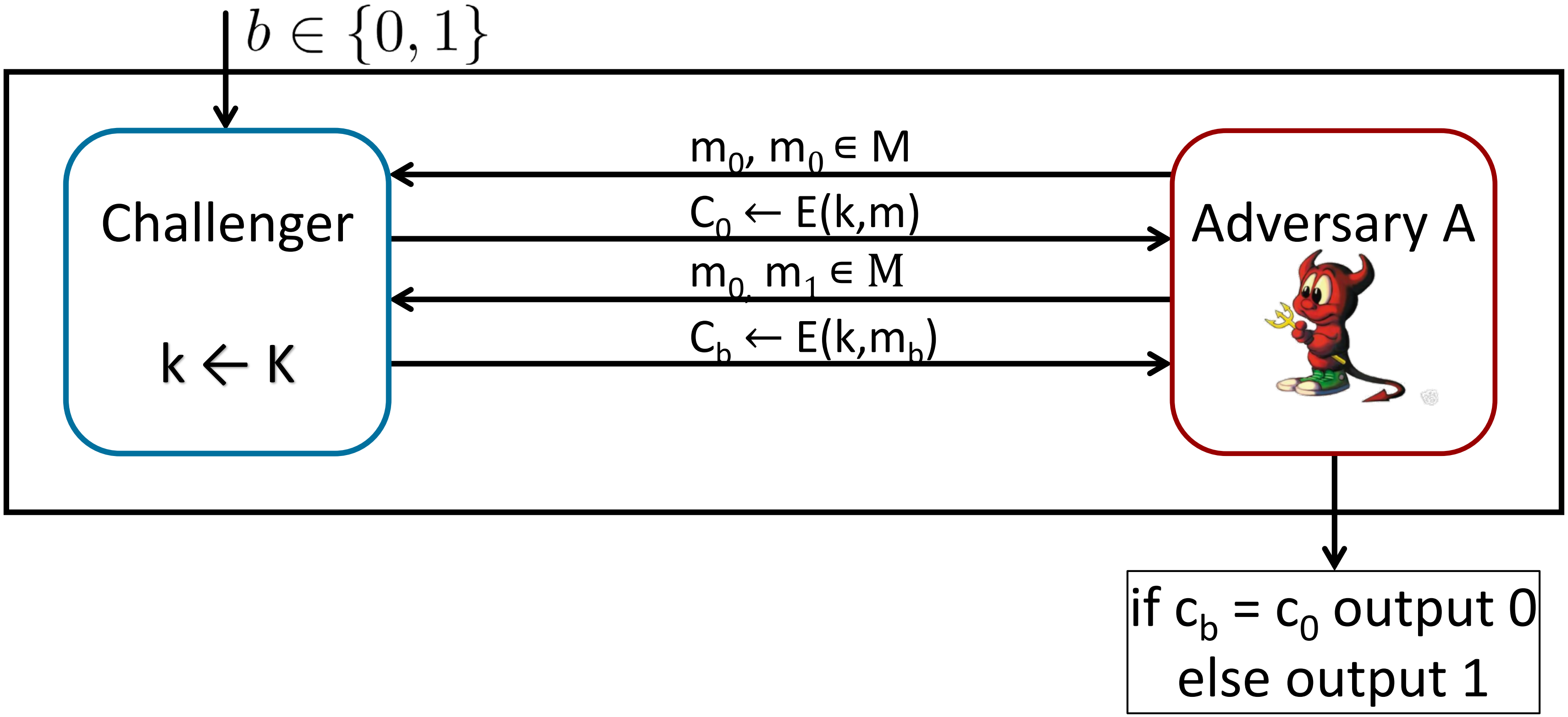
Ciphertext

Images from Wikipedia

Semantic security under Chosen Plaintext Attack (CPA)



ECB is not CPA secure



Semantic security under CPA

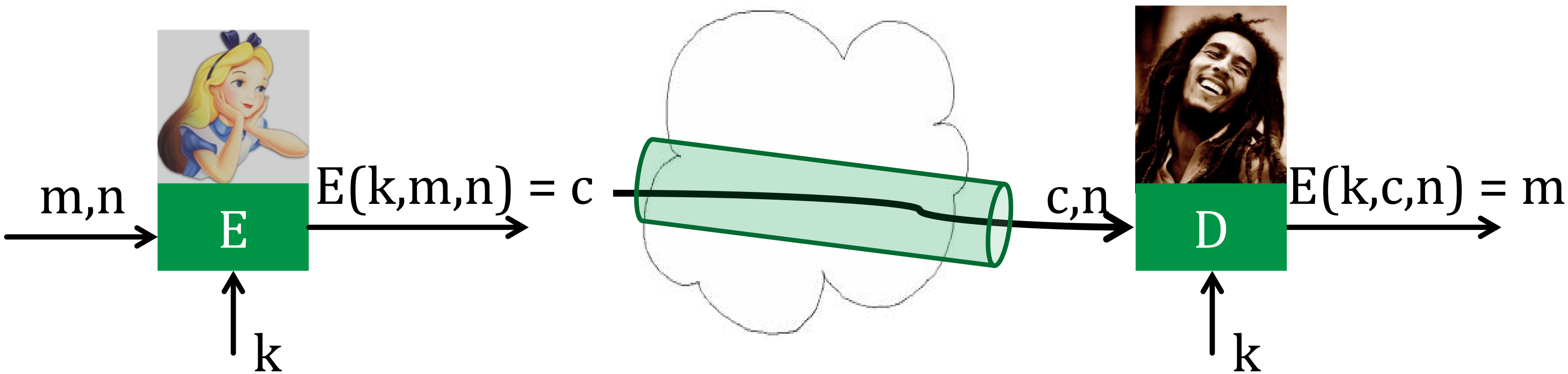
- Modes that return the same ciphertext (e.g., ECB) for the same plaintext are not semantically secure under a chosen plaintext attack (CPA) (many-time-key)

Encryption modes must be randomized

Nonce-based encryption

Nonce n: a value that changes for each msg.

E(k,m,n) / D(k,c,n)



(k,n) pair never used more than once

Nonce-based encryption

Method 1: Nonce is a counter

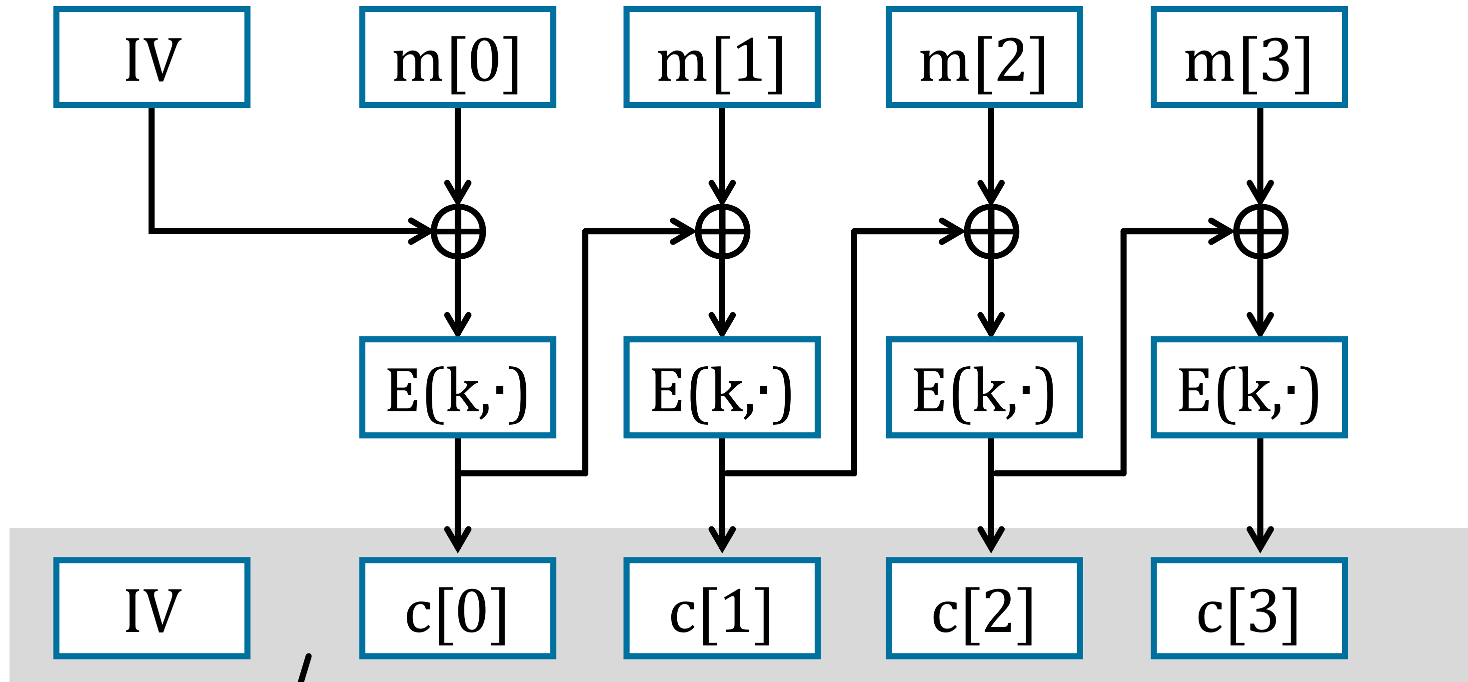
- Used when encryptor keeps state from msg to msg
- If decryptor has same state, nonce need not be transmitted (i.e., len(PT) = len(CT))

Method 2: Sender chooses a random nonce

- No state required but nonce has to be transmitted with CT

Cipher block chaining mode (CBC)

Let(E,D) be a PRP. $E_{CBC}(k,m)$: chose **random** IV ∈ X and do:



ciphertext

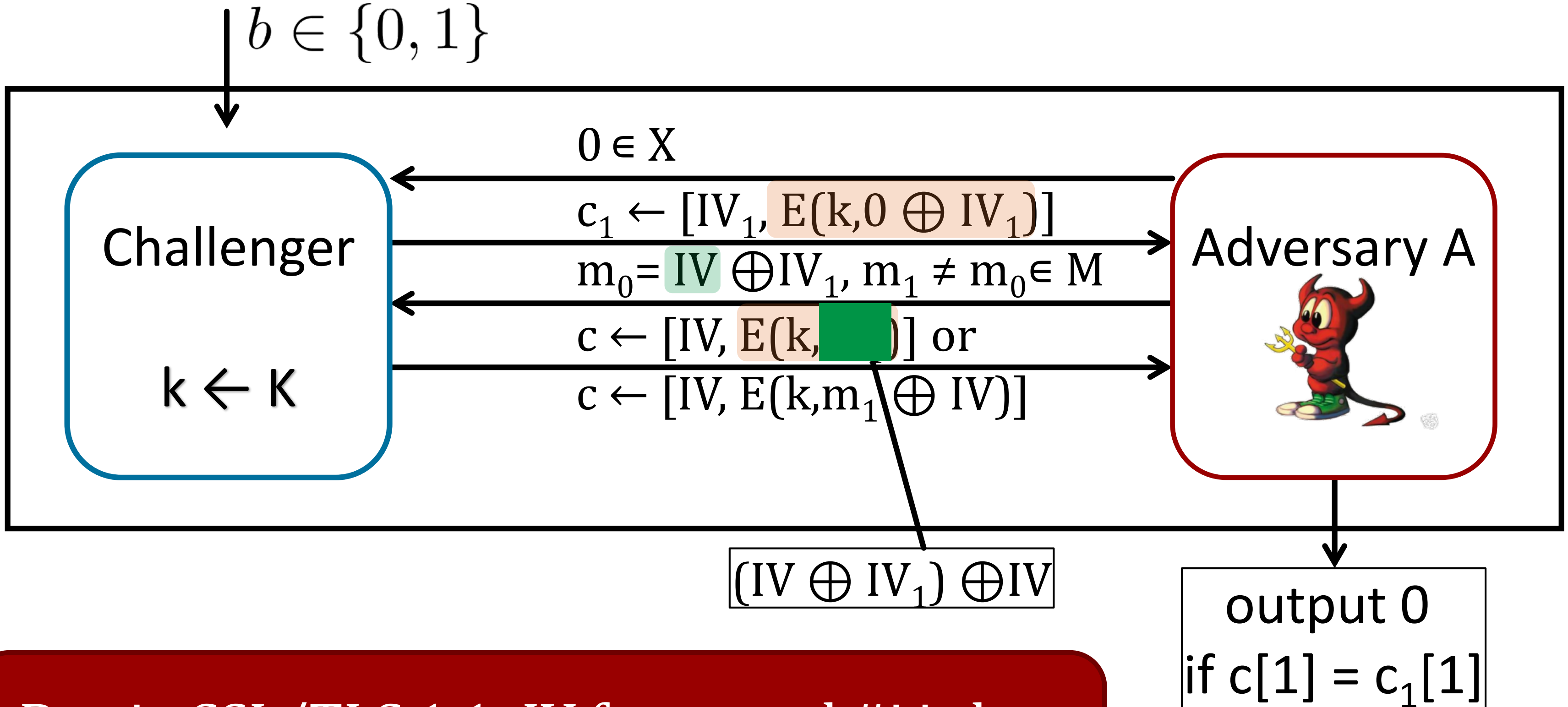
Decryption:

$c[0] = E(k, IV \oplus m[0]) \longrightarrow$

$m[0] =$

Attack on CBC with Predictable IV

Suppose given $c \leftarrow E_{CBC}(k,m)$ Adv. can predict IV for next msg.



Bug in SSL/TLS 1.1: IV for record #i is last CT block of record #(i-1)

Cipher block chaining mode (CBC)

Example applications:

1. File system encryption:

   use the same AES key to encrypt all files (e.g., loopaes)

2. IPsec:

   use the same AES key to encrypt multiple packets

Problem:

If attacker can <u>predict</u> IV, CBC is not CPA-secure

Summary

Block ciphers

- Map fixed length input blocks to same length output blocks
- Canonical block ciphers: 3DES, AES
- PRPs are effectively block ciphers
- PRPs can be created from arbitrary functions through Feistel networks
  - 3DES based on Feistel networks
  - AES based on substitution-permutation networks


Questions?

END

Linear and differential attacks [BS'89,M'93]

Given *many* inp/out pairs, can recover key in time less than 2^{56} .

<u>Linear cryptanalysis</u> (overview) :

let c = DES(k, m)

Suppose for random k,m :

$$\Pr\Big[\ m[i_1]\oplus\cdots\oplus m[i_r]\ \oplus\ c[j_j]\oplus\cdots\oplus c[j_v]\ =\ k[l_1]\oplus\cdots\oplus k[l_u]\ \Big] = ½ + \varepsilon$$

For some ε. For DES, this exists with

$\varepsilon = 1/2^{21} \approx 0.0000000477$

Linear attacks

$$\Pr\Big[\ m[i_1]\oplus\cdots\oplus m[i_r]\ \oplus\ c[j_j]\oplus\cdots\oplus c[j_v]\ =\ k[l_1]\oplus\cdots\oplus k[l_u]\ \Big] = \tfrac{1}{2} + \varepsilon$$

Thm: given $1/\varepsilon^2$ random (m, c=DES(k, m)) pairs then

$$k[l_1,\ldots,l_u]\ =\ \mathrm{MAJ}\Big[\ \ m[i_1,\ldots,i_r]\ \oplus\ c[j_j,\ldots,j_v]\ \ \Big]$$

with prob. $\geq 97.7\%$

$\Rightarrow$ with $1/\varepsilon^2$ inp/out pairs can find $k[l_1,\ldots,l_u]$ in time $\approx 1/\varepsilon^2$.

Linear attacks

For DES, $\varepsilon = 1/2^{21} \;\Rightarrow$

with 2^{42} inp/out pairs can find $k[l_1,\ldots,l_u]$ in time 2^{42}

Roughly speaking: can find 14 key "bits" this way in time 2^{42}

Brute force remaining $56-14=42$ bits in time 2^{42}

Total attack time $\approx 2^{43}$ ($<< 2^{56}$) with 2^{42} random inp/out pairs

Lesson

A tiny bit of linearity in S_5 lead to a 2^{42} time attack.

$\Rightarrow$ don't design ciphers yourself !!

Quantum attacks

Generic search problem:

Let $f: X \longrightarrow \{0,1\}$ be a function.

Goal: find $x \in X$ s.t. $f(x)=1$.

Classical computer: best generic algorithm time $= O(|X|)$

Quantum computer [Grover '96] : time $= O(|X|^{1/2})$

Can quantum computers be built: unknown

Quantum exhaustive search

Given m, c=E(k,m) define

$$f(k) = \begin{cases} 1 & \text{if } E(k,m) = c \\ 0 & \text{otherwise} \end{cases}$$

Grover $\Rightarrow$ quantum computer can find k in time $O(|K|^{1/2})$

DES: time $\approx 2^{28}$, AES-128: time $\approx 2^{64}$

quantum computer $\Rightarrow$ 256-bits key ciphers (e.g. AES-256)